Optimization of High-Frequency Trading Systems

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HFT

High-Frequency Trading

- Dynamics/signals timescale < seconds
- Demanding telecom/network & software/hardware engineering
- Machine Learning, Simulation, Experimentation
- Revenue-generating, agency execution

- microstructure
- ex, trading strategy: buy/sell for profit
- ex, execution system: fill an order for a customer

HFT: Technological Progress

- <1980’s: telephone, runners
- 1980’s/1990’s: computers, handhelds
- 2000’s: colo, fiber, FPGA
- 2010’s: microwave, mmwave, shortwave
- Technology is commoditized and widespread

- ongoing computerization of trading — like every other industry; steady progress
- called “program trading” in 1980s
- “electronic/algorithmic trading” in 1990s & early 2000s
- “high-frequency trading” since then
- roughly (by end of each decade):
    - 1980s: seconds
    - 1990s: millis
    - 2000s: micros
    - 2010s: nanos
- microwave: long distance, med. bandwitdth
- mmwave: short distance, high bandwidth
- shortwave: very long distance, very low bandwidth
- HFTs usually on the cutting edge of this progress
-

HFT: Automation of Trading “Stack”

- *Exchange: where trading occurs*
- Liquidity Provision: be available to trade
- Arbitrage: keep prices at fair values
- Execution: trade on behalf of a customer

- bottom three are HFT
- market “stack” is like:
  - (bottom) exchange, MM/arbitrageurs, execution services, investors (top)

- liquidity provision reduces the time to trade [like a used car dealer; easier than scanning posts on Craigslist]
- arbitrage makes sure assets are priced correctly, so you get a fair price when you trade [Do you own SPY or another ETF? Did you buy it at a fair price? How do you know?]

- You an investor (top of stack) goes to an exchange to trade and there’s a counterparty to trade with at a reasonable cost (liquidity), the asset are priced fairly (arbitrage), then the market is functioning well.

- execution algos takes work off of your hands; you hire a expert to do the grunt work and know the market [like a real estate agent helps you buy a house]; ex: (i) slowly work a large order, (ii) offer an interface that simplifies access to a large number of related markets (i.e., US equities)

Questions?

Optimization

A Trading Strategy

```
If signal > threshold: Buy Long

If signal < -threshold: Sell Short

If end of day: Liquidate and Stop
```

Rule set called a *policy*

threshold is a *parameter*

- Keep this example in mind as we go along

- Best threshold value depends on cost to trade, signal quality, how fast signal changes (decorrelates), cost to liquidate at EOD, and your definition of strategy quality (pnl, pnl - risk, etc.)
- How do you find the best threshold? That’s the subject of this talk…

Prediction	Control
independent estimations	sequence of decisions
known targets	no targets
error function	arbitrary: pnl, sharpe, …
signal weights	thresholds, weights, limits, …
(signal, response)	simulation, reality

- a prediction might be a useful component of a trading strategy, but the strategy is a controller
- prediction: ex: midprice 1 second from now, 1 minute from now, next trade price, etc.*
- “reward” for good decision might be given over time, while making other decisions; hard to determine *exactly which decisions responsible for pnl, etc.
- could make prediction a subproblem of controller (strategy) design; but not always clear what the target should be

Simulation …

- can evaluate sequence of decisions, long-term effects
- includes risk, liquidity
- cheap: run many sims

- long-term effects: ex., order has to sit in queue for long time
- cheap compared to trading

… vs. Reality

- But: Market reacts to our actions
- But: Hidden liquidity is … hidden
- But: Latencies complicated
- But: Exchange is complex
- But: Unknown unknowns

- trying to simulate a system with hidden state and complex dynamics
- matching engine processes our orders — even if they don’t get filled; takes time, changes market
- other traders (computers) see our orders/executions in public data and make different decisions than they would/could have
- Any visible queue can have hidden liquidity, too + dark pools = more hidden queues than visible; *most* queues are hidden (not most *shares*, but most queues)
- long-holding-time strategies (i.e., days) may treat all of these effects as a small, noisy cost; but they are significant for HFT where profits/share are on par with these costs
- latencies possible at every network node; latencies coupled to each other and likely also to signals
- exchange: what book is the exchange seeing right now? How are nearly-simultaneous messages reordered? How do complex order types *really* work?
- simulation useful for testing code quality, optimization methodology, operational risk assessment

Private Data?

- Model market's response to our orders, cancels
- Better, but not great:
  - What if we placed an order at a different time?
  - What if we *didn't* place an order at this time?

- incorporate private data (our orders and cancels) into simulation
- can build model of execution, but face
  - little private data to work with (relative to public data)
  - missing "counterfactuals" — what if we took a different action?

Wrong Objective

- Quality estimate in simulation != quality estimate in real trading
- similar to overfitting
- “Online-Offline Gap” [FB ML Field Guide]
- “Reality Gap” [Ev. Robotics]

- not unique to trading; pervasive in engineering

- similar to overfitting in SL problem: error function over your data sample != error function over full population
- but worse: your simulated dynamics might not even be a reasonable estimate of real dynamics; sometimes called “model bias” or overfitting of “tasks”, but less-clearly understood than SL overfitting (sample bias)

- Facebook Field Guide to ML [https://research.fb.com/videos/the-facebook-field-guide-to-machine-learning-episode-6-experimentation/]
- M. Palmer, D. Miller, An evolved neural controller for bipedal walking with dynamic balance [http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1016.6201&rep=rep1&type=pdf]

Questions?

Experiment

Prescription: experiment

Experimentation

- Measure *quality (Q)* of parameters by trading
- Measurement has a cost: loss, risk, opportunity
- Goal 1: Find highest-quality parameters
- Goal 2: Minimize cost of measurement

- "quality" could be pnl, pnl - risk, etc.; you decide
- Every day that you trade at a suboptimal parameter — even if you're making money — you're paying an opportunity cost. You've missed out on the extra money you would have made by trading at a better parameter setting.
- competing goals: Goal 1 says "more measurements", Goal 2 says, "fewer measurements"

Satisficing

- Guess parameters ("reasonable")
- Do they work? Be thankful and don't touch!

- satisfice = "satisfy" + "suffice" [https://en.wikipedia.org/wiki/Satisficing]
- go build another strategy: other instruments, other markets, etc.
- at HFTMM: scaled-up satisficing; ran many small strategies, turned off ones that lose money
- Why optimize? (i) lots more revenue available, (ii) System loses money w/o it, (iii) don't have experience/intuition to guess

A/B Test

- Compare two parameter sets / policies
- Call them “Policy A” and “Policy B”
- Ex: threshold=1 vs. threshold=2
- Ex: “JPM SOR” vs. “KCG SOR”

- can compare continuous parameter values or categorical, non-parameterized design decisions

A/B Test

- Trade A and B side-by-side for N days
- N determined by noise level and desired precision

$$N \approx \sigma^2 / \delta Q^2$$

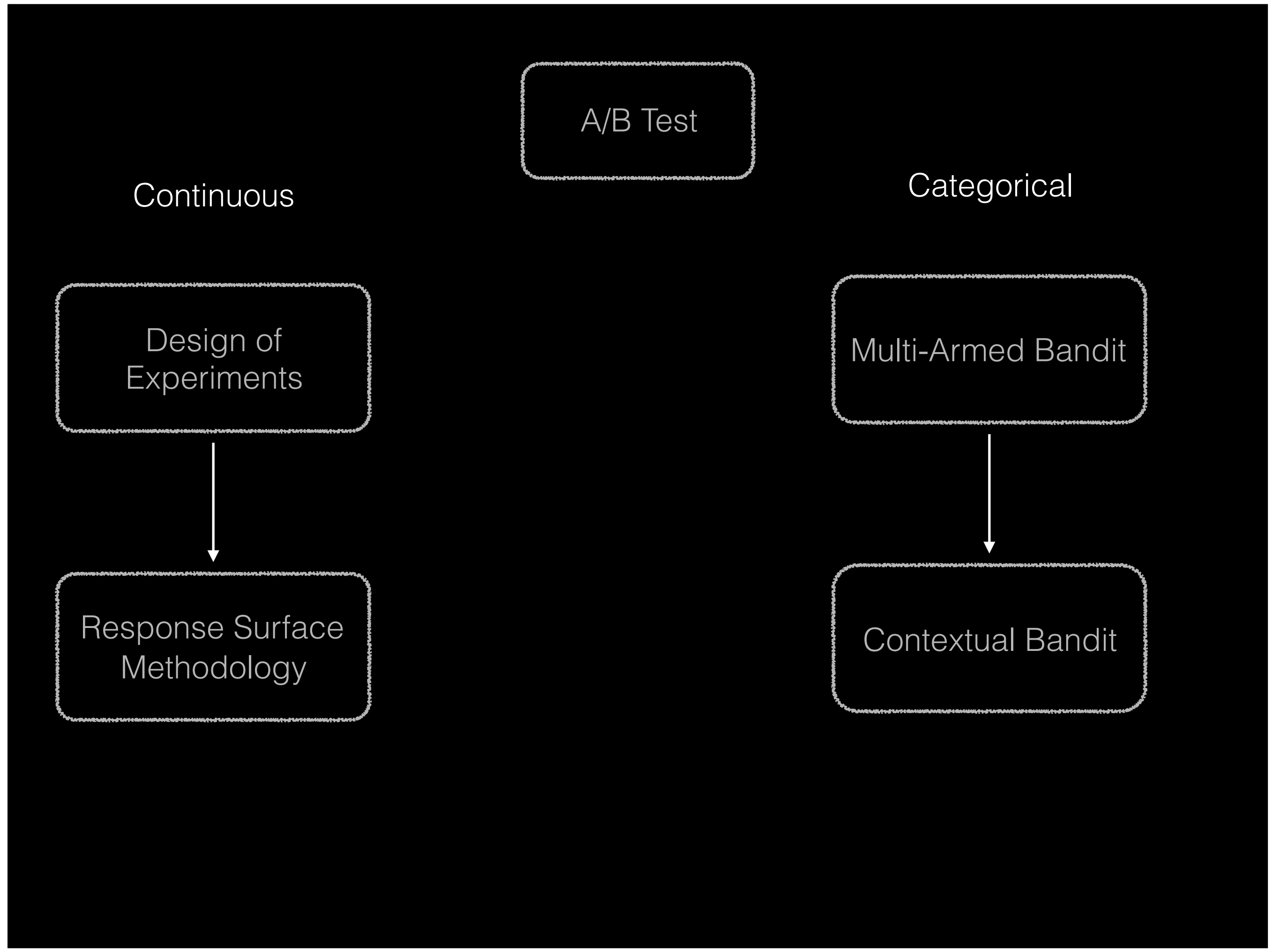
- sigma = std. dev. of a q measurement
- delta Q= smallest Q(A)-Q(B) you care to detect
- Ask, “Is B better than A?”
- EXAMPLE: VWAP Buy + VWAP Sell for each of A and B to test a change in execution signals, N = 1 day
- EXAMPLE: HFTMM in ~1000 stocks divided up into A & B sets to compare threshold (liquidity cost) settings, N = 10 trading days (two weeks)

- nice overview: https://towardsdatascience.com/data-science-you-need-to-know-a-b-testing-f2f12aff619a

Improving A/B

- Lower cost of measurements
- Evaluate more parameters, more settings

- Can we improve upon an A/B test?
- What if B is a *lot* better? Can't we stop early and lower the cost? [No, b/c your plan to deal with noise required N days.]
- What if we have more than two options to compare? A, B, C, …? A vs. B, then winner vs. C, then …   This could take a long time and be very expensive.
- queue of ideas to try can fill up quickly; want to service that queue quickly, too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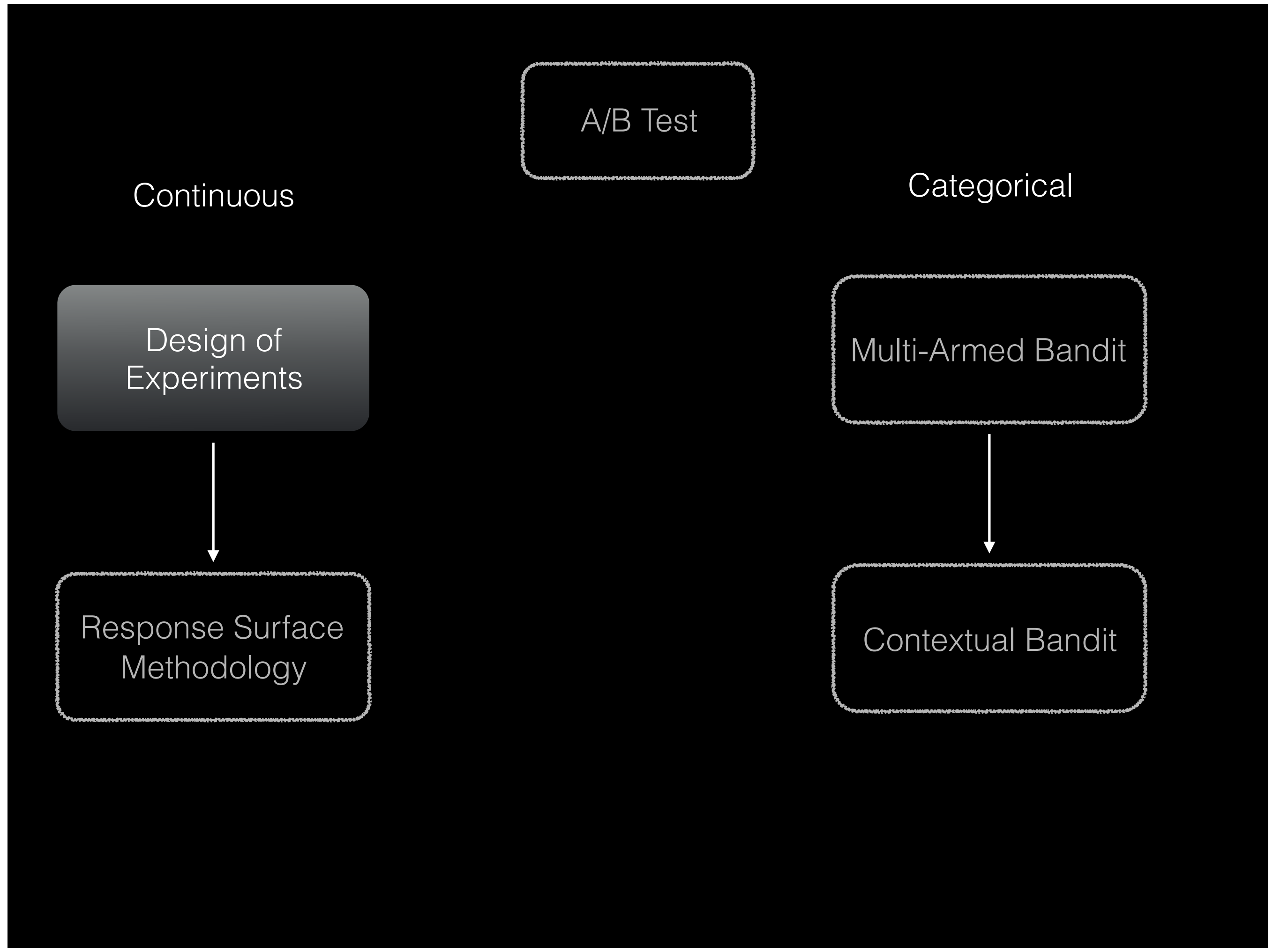
Continuous
Categorical
Design of Experiments
Multi-Armed Bandit
Response Surface Methodology
Contextual Bandit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Design of Experiments

- Evaluate multiple parameters' settings
- Choose which parameter values to measure to keep information high and cost low

- ex: threshold = 1, 2, 3, …
- *not* JPM vs KCG, however
- try to minimize # of experiments needs to evaluate settings of K parameters

Design of Experiments

Factorial

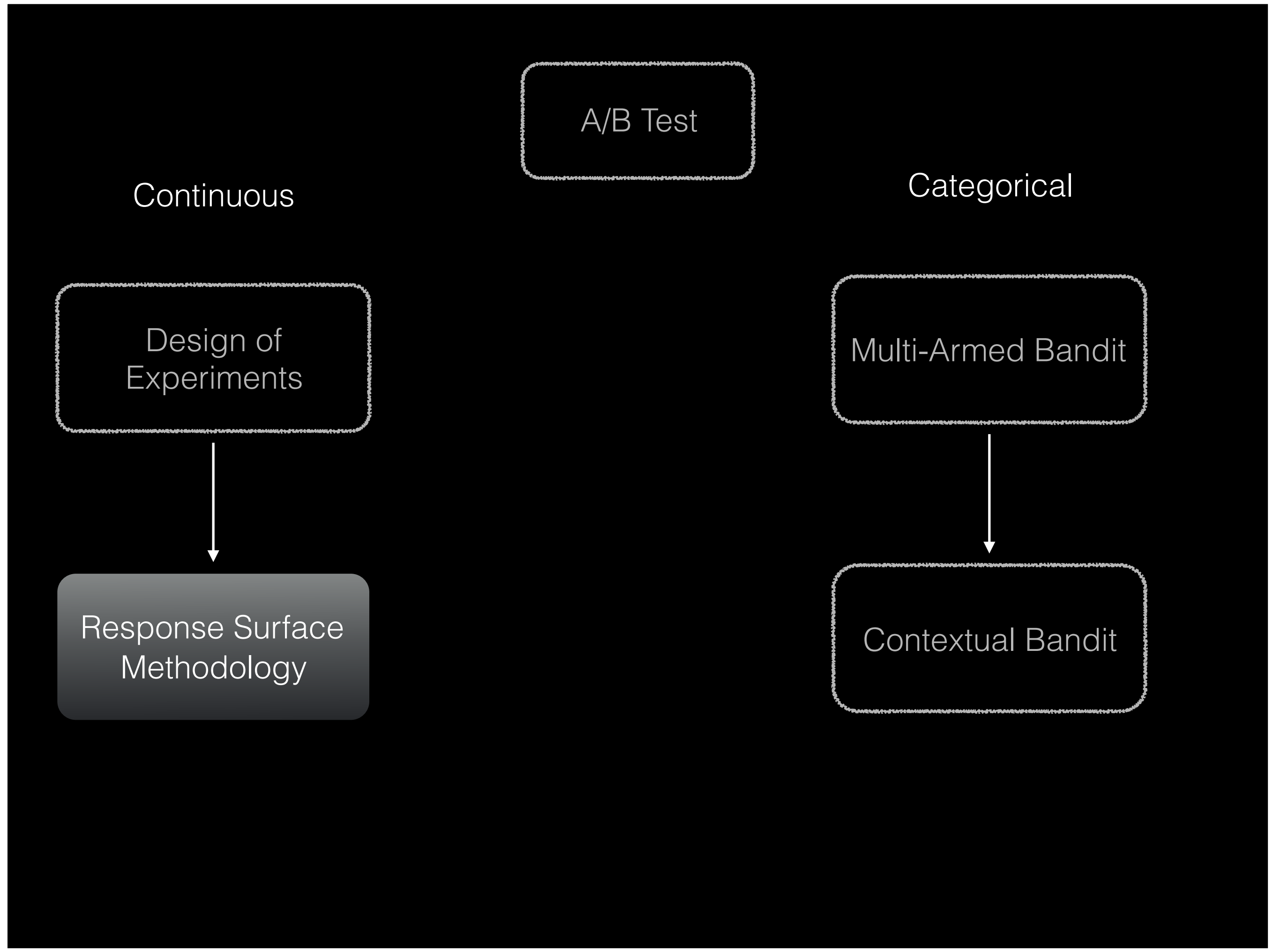
p1	p2	p3
-	-	-
+	-	-
-	+	-
-	-	+
+	+	-
+	-	+
-	+	+
+	+	+

Fractional Factorial

p1	p2	p3
-	-	-
+	+	-
+	-	+
-	+	+

- Factorial: all combinations, 2^n measurements
- Fractional Factorial: Try to assess each parameter independently by removing pair-wise correlation; (only measure 1st and 2nd order effects)
- avoid: “Hey! When I increased p1, quality improved!” “But when you increased p1 you also increased p2. So which parameter is responsible for the improvement?”
- Fewer measurements = lower cost
- EXAMPLE: MM strategies, would run full-factorial designs on two parameters and fractional factorial designs on three parameters
- more complicated with more parameters; There are tables online. :)
- What about values between - and +? Can we be more precise? Can we handle more parameters without a large number of experiments?

NIST Engineering Handbook https://www.itl.nist.gov/div898/handbook/pri/section1/pri1.htm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Response Surface Methodology

- Model (regress) quality vs. parameters from D.O.E data
- *Infer* the best parameters from model!
- Verify/Improve: D.O.E. around inferred-best

- Model (regress) quality vs. parameters
- The "best" parameters likely won't be in the data set.
- Re-center the measurements around the inferred-best. Then take measurements to verify your inference.
- Repeat if desired until your inferred-best stops changing.
- This is an iterative (manual) optimization routine
- EXAMPLE: Designed intraday strategy, ~1000 stocks, using simulation. Ran with various values of a threshold parameter, modeled quality vs. parameter, and set to inferred-best value. Did not iterate, however.

“Automated RSM”

Model (regress) response surface, Q(params)

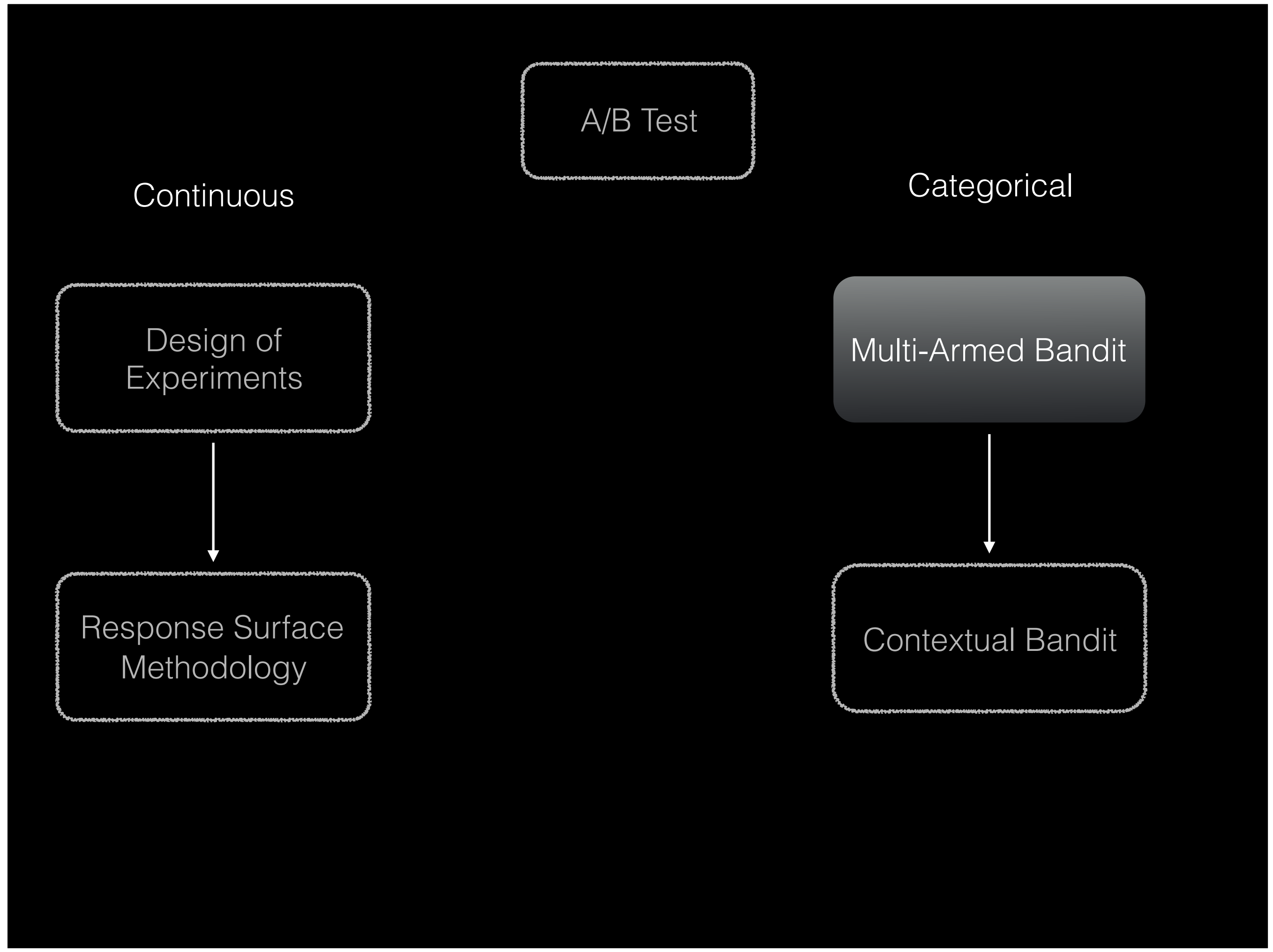
Maximize Acquisition Function, ex: Q + stderr Q

Run experiment

Repeat

- many algorithms; Kriging, Bayesian Optimization, Efficient Global Optimization, Surrogate-function Black-Box optimization methods
- (3) tries to optimally trade off the need to collect more data (to build a better model) which has a cost with the desire to trade at the optimal parameters; aka “exploration vs. exploitation”
- exploitation => higher revenue now; exploration => higher revenue in the future
- accounts for noise / uncertainty in each measurement, so each trading day can use a new experiment design; all data are combined optimally into RSM

Efficient Global Optimization of Expensive Black-Box Functions http://www.ressources-actuarielles.net/EXT/ISFA/1226.nsf/0/f84f7ac703bf5862c12576d8002f5259/$FILE/Jones98.pdf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Multi-Armed Bandit Problem

- "one-armed bandit" == slot machine
- MAB: K arms, each with different, noisy payout
- Strategy to optimize total payout?

- MAB is a problem definition
- "MAB methods" are ways to solve that problem
- arms are parameter settings
- K=2 arms == a more efficient A/B test
- MAB cares about measurement cost
- MAB handles multiple choices (not just two)

Multi-Armed Bandit Methods

1. Pull each arm several times
   Q(arm) = mean(arm quality measurements)
   Thereafter only pull highest-Q arm

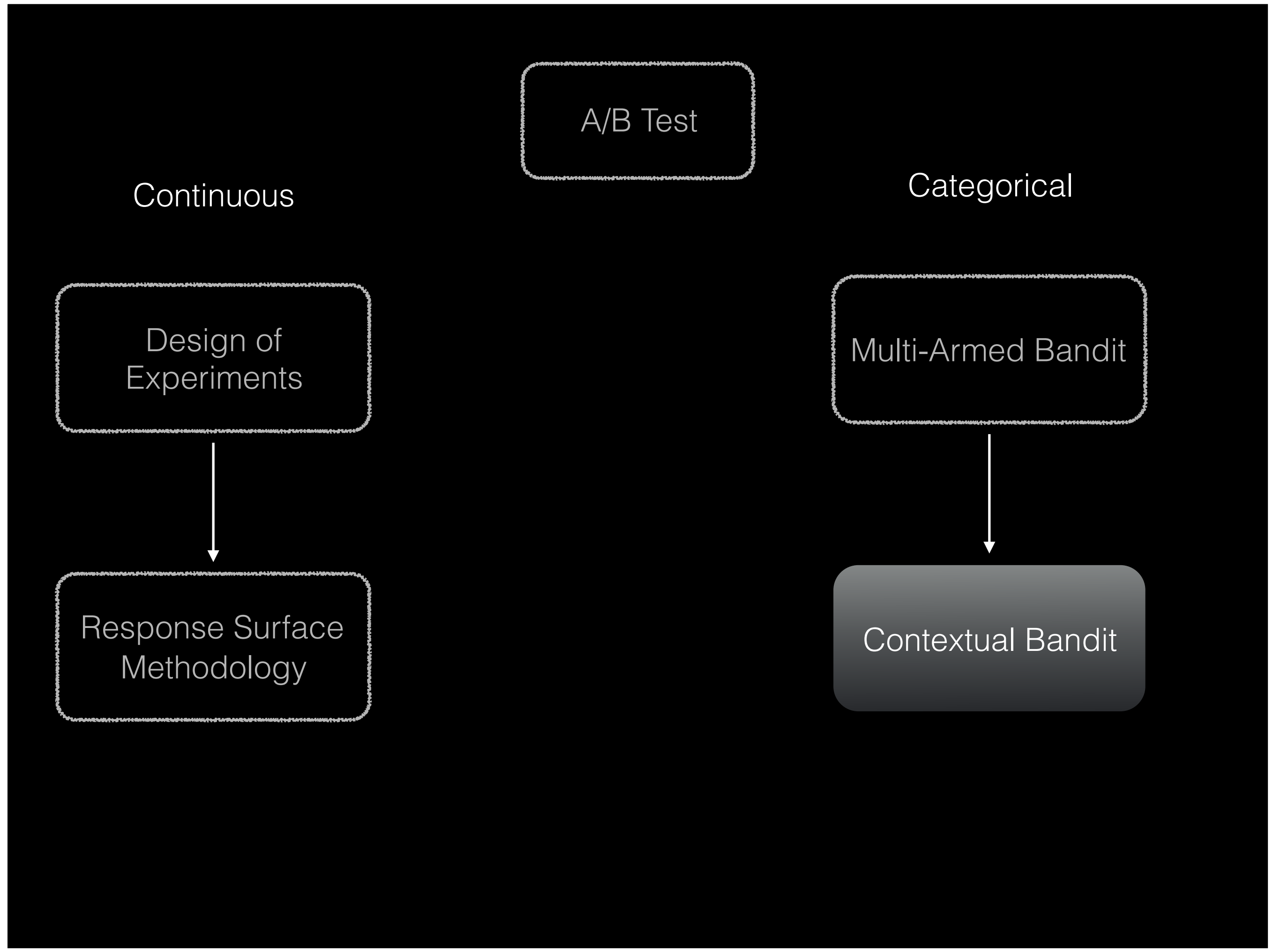
2. p=.9: pull highest-Q arm
   p=.1: pull random arm

3. Pull arm with maximal "Q + stderr(Q)"

- (1) spends a lot of time measuring, but ultimately pulls the best
- (2) (eps-greedy) "explores" 10% of time to improve estimates, but usually (90% of time) pulls the one we think is best; but never stops exploring
- (3) (UCB1, if stderr is modified a bit) expression makes exploration vs. exploitation explicit; adds more samples to the noisier estimates (more efficient exploration); eventually stops exploring (more efficient exploitation);

- EXAMPLE: HFTMM; would run ~10,000 arms each day dropping worst arms each night and adding new arms each morning; arm design initially manual, but grew more and more systematic (and higher-parameter) over time

Multi-Armed Bandit Algorithms and Empirical Evaluation https://cs.nyu.edu/~mohri/pub/bandit.pdf



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Contextual Bandit

- context (aka. state) == signals, time of day, product traded, etc.
- Q(arm, context) = regression model
- Fit model from measurements so far
- Decision like MAB: Q + stderr(Q)

- Follow same rules as MAB — 90%/10% or maximal mean+se, except means are replaced by conditional means, i.e. model's prediction of arm quality
- EXAMPLE: Execution Router: four brokers to route orders to; model slippage of parent order based on broker, time of day, product, other signals; rebuild model every night to "learn" from the day's activity
- EXAMPLE: ad-hoc in HFTMM; choice of strategies to run was conditioned on time of day, market volume/volatility

Learning for Contextual Bandits (slides) http://hunch.net/~exploration_learning/main.pdf

A Contextual Bandit Bake-off https://arxiv.org/abs/1802.04064

THOMPSON SAMPLING WITH THE ONLINE BOOTSTRAP https://pdfs.semanticscholar.org/d623/c2cbf100d6963ba7dafe55158890d43c78b6.pdf

Questions?

Reinforcement Learning

- modern ML methods; “AI”, even

Reinforcement Learning

- SL : Prediction :: RL : Control
- RL Goals:
    - automate engineering of controllers
    - increase controller sophistication

- SL: predict outcome from signals; learn from examples (face recognition, translation)
- RL: decide actions based on signals; learn from experience (Go, robots)
- sophistication: more signals, more actions, more complex sequences of actions

Sutton & Barto, Reinforcement Learning: An Introduction, http://incompleteideas.net/book/bookdraft2017nov5.pdf
Robot Hand https://arxiv.org/abs/1703.06907

RL Methods

- Evolutionary Algorithms (DeepGA, OpenAI-ES)
- Policy Gradient (PPO, DDPG)
- Value-based (DQN)
- Model-based (ME-TRPO, World Models)

- lots more, too
- RL: flexible, parameterizes models; automated optimization of parameters

DeepGA: https://arxiv.org/pdf/1712.06567.pdf

OpenAI-ES: https://arxiv.org/pdf/1703.03864.pdf

PPO: https://arxiv.org/abs/1707.06347

DDPG: https://arxiv.org/abs/1509.02971

DQN: https://storage.googleapis.com/deepmind-media/dqn/DQNNaturePaper.pdf

ME-TRPO: https://arxiv.org/abs/1802.10592

World Models: https://arxiv.org/abs/1803.10122

Sample Efficiency

- Most methods run (too) many experiments to run in production
- Maybe:
  - Model-based methods
  - Meta-learning

- 1MM - 100MM “experiments” (simulation runs in published papers)
- MBRL: experiments collect data, optimization happens in simulation

- MBRL: http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_9_model_based_rl.pdf
- Meta: http://www.cantab.net/users/yutian.chen/Publications/ChenEtAl_NIPS16Workshop_L2LBlackBoxOptimization.pdf

Model-Based RL

- Learn the simulator from data
- Optimize controller in simulation
- Run controller to collect more data
- Repeat

- maybe optimize controller maximize pnl as well as collect more data to improve sim

Meta RL

- Construct an optimizer customized for:
  - Your controller and your environment
- Optimize the optimizer in simulation
- Optimize the controller by experimentation

- custom optimizer is flexible (lot of parameters)
- (one) objective is to optimize controller with *very* few experiments

Questions?